Enhancing Finite-Difference Time-Domain Analysis of Dielectric Resonators Using Spectrum Estimation Techniques

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Abstract *In this paper we discuss the use of digital filtering and spectrum estimation techniques for improving the efficiency of the FD-TD algorithm in solving eigenvalue problems. It is demonstrated by means of numerical results that the efficiency of the FD-TD method for dielectric resonator analysis has been improved at least one order of magnitude. This new result makes it possible for the FD-TD method to be a practical designing tool for the dielectric resonator analysis.*

I. Introduction

The optimization of the performance of resonators in microwave circuits requires accurate and efficient methods for calculating the resonant frequencies and the spatial distributions of the field. Various methods have been developed to study the resonant frequencies of resonant structures. Most of them, such as; the mode matching method, integral equation method, and finite element method, are carried out in the frequency domain [1].

The finite-difference time-domain (FD-TD) method has been widely used for solving electromagnetic problems. Resently, it has been used to solve eigenvalue problems associated with resonator structures [2],[3] and to calculate critical parameters for complex microstrip antennas [4]. All of these results have shown the FD-TD method to be a very powerful tool for eigenvalue analysis, primarily because of two desirable attributes. First, it can be applied to problems exhibiting a complex structure which may be very difficult to solve using other analytical or numerical methods. Second, only one computation is required to get the frequency domain results over a large frequency spectrum. However, this method has one significant drawback, which is that it requires a very long computation time for extracting the resonant frequencies from the FD-TD results; for example, in the case of the problem discussed in [2], the time iteration N has to be as large as $N = 2^{16}$.

The main purpose of this paper is to introduce the use of digital filtering and modern spectrum estimation techniques with the FD-TD method, as a means for overcoming above limitation. By using numerical results, it will be shown that modern spectrum estimation techniques can reduce the time taken to solve a problem, such as that discussed in [2], by one order of magnitude, without any loss of accuracy in calculating the resonant frequencies.

II. FD-TD Method for Resonator Analysis

For ease of description, the method is described by referring to the generalized cylindrically shaped dielectric resonator (DR) in Fig. 1. To start the computation, the initial electric and magnetic fields are set to zero throughout the grid, except at one selected point. Here the electric field is set to 1. This unit impulse source will excite a large number of modes. Using the FD-TD algorithm [5], Fig. 2a gives the computed electric field in the time domain at the point of observation. The resonant frequencies can be obtained by taking the Fourier transform of the computed time domain response. The field distribution for any particular frequency can be obtained by performing Fourier transform at each point in the computation domain at that frequency. With the objective of getting more accurate estimates of the resonant frequency and field distribution than that have been obtained in [2], the following procedure is put forward.

The procedure to be followed is based on the signal analysis of the time domain results obtained using the FD-TD method. In this section, it is assumed that the sequence length of the FD-TD result $\{x(n)\}$ is very long, where $x(n)$ is one of the field components. The procedure that has been suggested here is to do the numerical integration of Fourier Transform of $x(t)$ directly in the interested frequency band

$$\begin{aligned} X(f) &= \int_0^\infty x(t)\exp(-j2\pi ft)\mathrm{d}t \\ &\approx \int_0^{N\Delta t} x(t)\exp(-j2\pi ft)\mathrm{d}t \\ &\approx \sum_{n=0}^{N-1} x(n)\exp(-j2\pi fn\Delta t)\Delta t. \end{aligned} \quad (1)$$

The advantage of this method is that it removes ambiguities sometimes encountered with the discrete Fourier

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Transform, due to narrowband signal components with center frequencies that lie in the gaps between the $N/2$ frequency points evaluated with the DFT. It will be shown in the following that, when FD-TD method is used for resonator analysis, the time domain results are signals which consist of many narrowband signal components. The accuracy of calculating the spectral peaks, i.e. the field distribution, is also enhanced by using (1).

The efficiency of calculating (1) can be greatly improved by using the following method. Instead of using the original sequence $\{x(n)\}$ obtained using the FD-TD result, a new sequence $\{x_1(n)\}$ is used in (1), which is obtained by desampling the $\{x(n)\}$ at a certain rate. The desampling rate is determined by the ratio of the half sampling frequency $f_s/2$ to the maximum frequency f_{max} of the long sequence $\{x(n)\}$. Because $\{x_1(n)\}$ is much shorter than the original sequence $\{x(n)\}$, the time required to analyze the new time domain sequence can be greatly reduced, with no reduction in the accuracy of the result. The theory which supports this treatment is the popular Nyquist sampling theorem. For dielectric resonator analysis, the maximum frequency of the FD-TD time domain result is equal to about $f_{max} = v/(2\Delta h)$, where v is the speed of light in the largest dielectric constant material and Δh is the space step (where a uniform grid is assumed) [5].

In Fig. 2b, the solid line gives the result obtained by applying (1) to the long sequence, and the dashed line gives the result obtained by applying (1) to the decimated sequence $\{x_1(n)\}$. The two results are exactly the same and therefore overlap in Fig. 2b.

III. Use of Digital Filtering and Modern Spectrum Estimation Techniques with FD-TD Method

The objective of this section is, based on a much shorter sequence $\{x_2(n)\}$ obtained directly from the output of FD-TD algorithm, to use digital filtering and modern spectrum estimation techniques to extract the resonant frequencies of the dielectric resonator. Suppose $\{x_2(n)\}$ is the sequence consisting of the first two thousand data points in $\{x(n)\}$. The DFT spectrum of $\{x_2(n)\}$ is shown in Fig. 3a. After desampling $\{x_2(n)\}$, using a desampling rate of $(f_s/2)/f_{max}$, which is about 10, we get a sequence $\{x_3(n)\}$ whose DFT spectrum is shown in Fig. 3b. Because we are interested in the lower frequency band, we further process the signal $\{x_3(n)\}$ by using a decimating filter to get $\{x_4(n)\}$, whose DFT spectrum is shown in Fig. 3c. In order to improve the accuracy of estimating the resonant frequencies of the first few modes, we further process $\{x_4(n)\}$ with a low-pass filter and get $\{y(n)\}$, whose DFT spectrum is shown in Fig. 3d. In the next phase of the work we carry out a search for a good high resolution spectrum estimator, with which to extract the resonant frequencies from the data $\{y(n)\}$.

From the behavior of the spectrum of $\{x(n)\}$, based on the results given in Fig. 2b, it seems reasonable to assume that $\{y(n)\}$ is composed of sinusoidal components. One of the best kinds of methods for estimating the frequencies of sinusoidal components is the multiple signal classification (MUSIC) method [6],[7]. This method belongs to the eigendecomposition-based class of super-resolution spectrum estimation methods. The term "super-resolution" refers to the fact that this class of methods have the ability to surpass the limiting behavior of classical Fourier-based methods. There are a number of reasons for our choosing the MUSIC algorithm from amongst this class of methods. These are: (i) it is easy to implement, (ii) it provides good performance, (iii) it is used as a bench mark in the field of signal processing, and (iv) it provides a good introduction to modern spectrum estimation.

IV. Numerical Results

Using the MUSIC algorithm, the signal $\{y(n)\}$ which was obtained after filtering processing in the last section is analyzed. The result is shown in Fig. 4. The dashed line was obtained by applying a Fourier transform (1) to a very long FD-TD sequence, corresponding to 20000 time iterations in the FD-TD algorithm. The dotted curve gives the result from Fourier processing (1) of the first two thousand points in the former sequence. This shortened sequence corresponds to 2000 time iterations in the FD-TD algorithm. From this curve we see that, for short data records, the resonant frequencies cannot be accurately estimated using the Fourier transform. Biases occur in the locations of the first and fourth resonant frequencies and the second and third resonant frequencies are missing altogether. The solid line gives the result of application of digital filtering and the MUSIC spectral estimation technique to the shorter data. In the MUSIC algorithm, the data length of $\{y(n)\}$, K, was equal to 100, the order of the correlation matrix, $M+1$, was determined by the relation $M = 2K/3$. The accuracy of the method increases with increasing M. However, $M+1$ should not be larger than the number of data points. The choice for the order of the signal subspace, p, is based on the eigenvalue spectrum of the estimated correlation matrix. For our example, p was equal to 21. When p was changed from 21 to a higher value, we still got accurate frequency estimates. This suggests that the method is robust. Comparing the solid and dashed lines, it is seen that the same order of accuracy is obtained in the resonant frequency estimation by applying signal processing and spectral estimation to a short data set as that obtained by applying a Fourier transform to a much longer data set.

V. Conclusions

There are two main results coming from the present study of the FD-TD method. Digital filtering and modern spectrum estimation techniques were successfully incorporated with the FD-TD method as a means of improving its efficiency for carrying out eigenvalue analysis. The efficiency and validity of the method are demonstrated using both numerical and measured results. Another relatively new spectrum estimation method, which is called Thomson's multiple-window-method (MWM) [8], was also tested with FD-TD data. The same good frequency estimates were obtained using MWM. The second main outcome of this research was the application of signal analyses to the time domain data obtained using the FD-TD algorithm. It has been shown that the FD-TD time domain signal for dielectric resonator analyses is much over sampled. The data that needs to be retained for later processing can be greatly compressed, without degrading the accuracy of the analysis. This conclusion is valid when the FD-TD method is used to analyze microstrip components and antennas. In these latter cases, the maximum frequency of time domain result, f_{max}, which sets the desampling or compressing rate, is not determined by the cutoff frequency of the FD-TD algorithm itself, but rather by the maximum frequency of the excitation gaussian pulse. According to our experience, for the analysis of microstrip antennas and components [4], the data from the FD-TD results can be compressed by at least one order of magnitude. The above two conclusions are applicable to other time domain methods, such as the Transmission Line Matrix method.

In conclusion, it should be mentioned that signal processing and spectrum estimation techniques can greatly improve both the capability and the efficiency of the time domain methods. This point has been reinforced by several papers [9-14], where the authors have to greater or less degree drawn on signal processing techniques to improve the performance of their numerical algorithms.

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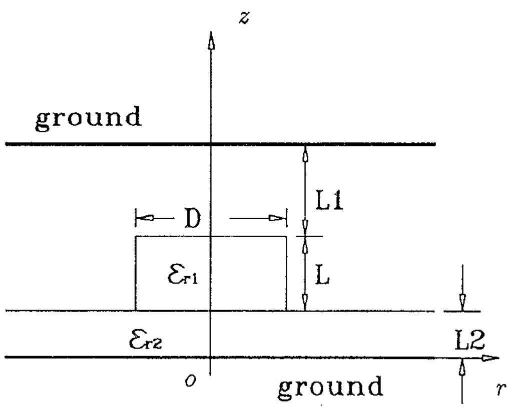
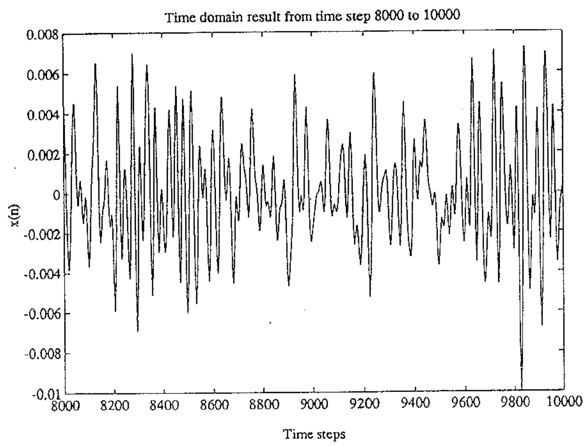
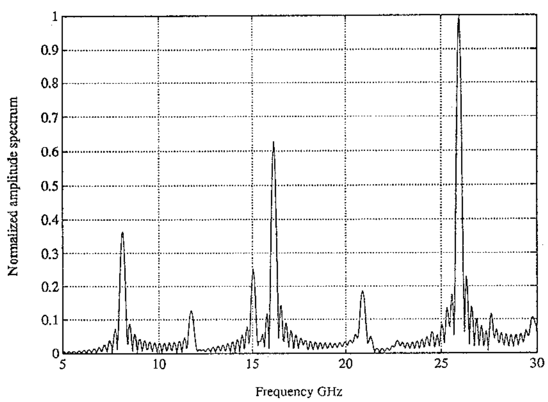


Figure 1 A generalized cylindrical shaped dielectric resonator.



(a)



(b)

Figure 2 (a) Time domain result directly obtained from the FD-TD algorithm; (b) Normalized amplitude spectrum of Fourier transform of long sequence and desampled shorter sequence.

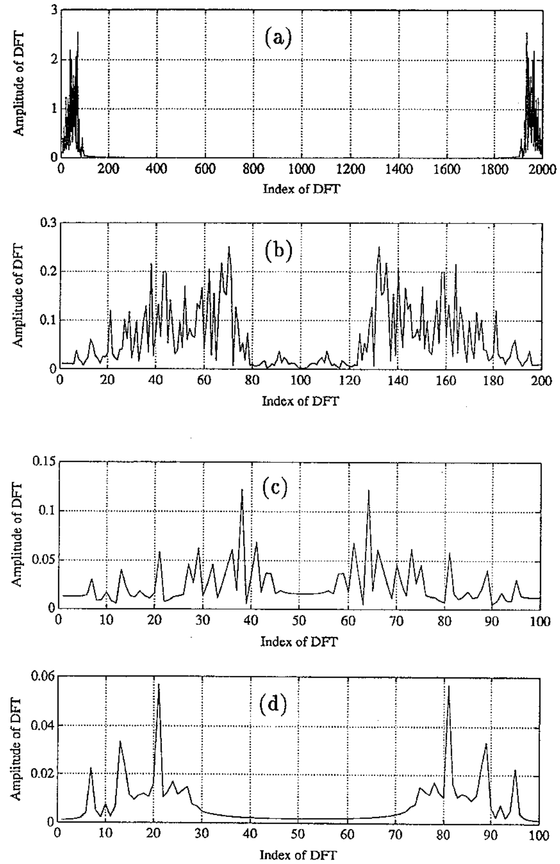


Figure 3 Digital filtering processing of the FD-TD time domain result. (a-d) DFT spectrums.

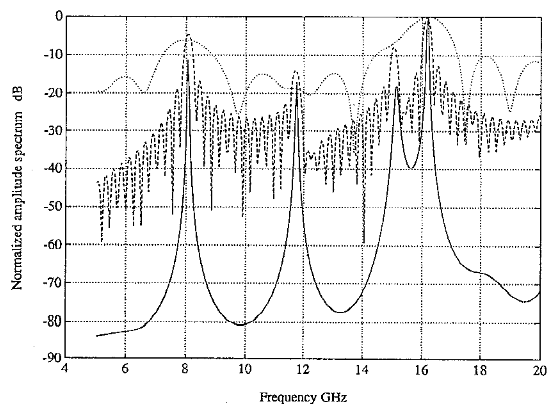


Figure 4 Resonant frequency estimation using different methods. Dashed line: 20000 iterations and using Fourier integration Eq. (1); Dotted line: 2000 time iterations and using Fourier integration Eq.(1); Solid line: Using 2000 time iterations in FD-TD algorithm and using digital filter processing and MUSIC method.